VACUUM STRUCTURE IN QCD WITH QUARK AND GLUON CONDENSATES

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Abstract

We consider here the vacuum structure in QCD with both quark and gluon condensates and a variational ansatz for the ground state. The method is nonperturbative using only equal time algebra for the field operators. We then find that a constrained energy minimisation of the Hamiltonian leads to a QCD vacuum with both quark and gluon condensates for $\alpha_s > \alpha_c = 0.62$. Pion decay constant and the charge radius of the pion seem to fix the QCD coupling constant α_s as 1.28. The approach opens up possibilities of relating the mysterious vacuum structure with common place hadronic properties.

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I. INTRODUCTION

It is now believed that quantum chromodynamics (QCD) is the correct theory for strong interaction physics of quarks and gluons. At low energies, however, the coupling constant in QCD becomes large leading to breakdown of the perturbative calculations. In this regime, the vacuum structure is also known to be nontrivial [1] with nonzero expectation values for quark and gluon condensates [2]. Instability of QCD vacuum with constant chromomagnetic field or with vortex condensate formation has been studied since quite some time with a semiclassical approach [3]. QCD vacuum has also been studied with gluon or glueball condensates [4,5] as well as with nonperturbative solutions to Schwinger Dyson equations [6]. Further, a nontrivial vacuum structure with quark condensates in Nambu Jona Lasinio type of models [7] has been seen to be consistent with low energy hadron physics. It is therefore desirable to examine the vacuum stucture in QCD with *both* quark and gluon condensates.

We had proposed earlier a variational method which is nonperturbative with an explicit structure for the QCD vacuum. This has been applied to the case of gluon condensates for vacuum structure in $SU(3)$ Yang-Mills fields demonstrating the instability of perturbative vacuum when coupling is greater than a critical value [8]. Same methods have been applied to study the vacuum structure with quark condensates in QCD motivated phenomenological effective potential models [9] and in Nambu-Jona-Lasinio model [10]. For the ground state or vacuum this has been achieved through a minimisation of energy density, free energy density or thermodynamic potential depending on the physical situation. For the chiral symmetry breaking we had also considered a simple ansatz of taking the perturbative quarks having a phenomenological gaussian distribution in the nonperturbative vacuum [11]. This appeared to describe a host of low energy hadronic properties as being related to the vacuum structure associated with chiral symmetry breaking. However, here no energy minimisation has been attempted. In the present paper we shall analyse the vacuum structure in QCD with both quark and gluon condensates and discuss its stability as opposed to taking *only*

gluon condensates [2–6,8] or *only* quark condensates [7,9,10].

As earlier, we shall take specific forms of condensate functions for quarks and gluons to describe a trial state for destabilised vacuum. Such an ansatz necessarily has limited dynamics. We shall circumvent this partially with the constraints that the SVZ parameter $\frac{\alpha_s}{\pi} < G^a_{\mu\nu} G^{a\mu\nu} >$ and the pion decay constant c_π shall be correctly reproduced as experimentally known. The condensate functions still contain two parameters, over which energy is minimised. We then note that for α_s greater than a critical coupling α_c, perturbative vacuum destabilises with nonvanishing condensates in *both* quark and gluon sectors.

We organise the paper as follows. In section II, we briefly recapitulate quantisation of QCD in Coulomb gauge and give an explicit construct for the nonperbutative vacuum with quark and gluon condensates. The ansatz for the QCD vacuum is similar to BCS ansatz of Cooper pairs in the context of superconductivity. In section III we consider the stability of such a trial state through an energy minimisation where pion decay constant and SVZ parameter are taken as constraints as above and discuss the results. In section IV we summarise our conclusions. We also notice that for $\alpha_s \simeq 1.28$, the ansatz function of Ref. [11] along with the correct charge radius of the pion is reproduced.

The method considered here is nonperturbative as we shall be using the equal time quantum algebra for the field operators, but is limited by the choice of the ansatz functions. The technique has been applied earlier to solvable cases [12] to examine its reliability and to ground state structure for electroweak symmetry breaking and cosmic rays [13] and to nuclear matter, deuteron or quark stars [14].

II. VACUUM IN QCD WITH QUARK AND GLUON CONDENSATES

The QCD Lagrangian is given as

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{matter} + \mathcal{L}_{int}, \tag{1}$$

where

$$\mathcal{L}_{gauge} = -\frac{1}{2}G^{a\mu\nu}(\partial_\mu W^a{}_\nu - \partial_\nu W^a{}_\mu + gf^{abc}W^b{}_\mu W^c{}_\nu) + \frac{1}{4}G^a{}_{\mu\nu}G^{a\mu\nu}, \tag{2a}$$

$$\mathcal{L}_{matter} = \bar{\psi}(i\gamma^\mu \partial_\mu)\psi \tag{2b}$$

and

$$\mathcal{L}_{int} = g\bar{\psi}\gamma^\mu \frac{\lambda^a}{2}W^a_\mu\psi, \tag{2c}$$

where $W^a{}_\mu$ are the SU(3) colour gauge fields. We shall quantise in Coulomb gauge [15] and write the electric field $G^a{}_{0i}$ in terms of the transverse and longitudinal parts as

$$G^a{}_{0i} = {}^TG^a{}_{0i} + \partial_i f^a, \tag{3}$$

where f^a is to be determined. We take at time t=0 [8]

$$W^a{}_i(\vec{x}) = (2\pi)^{-3/2}\int \frac{d\vec{k}}{\sqrt{2\omega(\vec{k})}}(a^a{}_i(\vec{k}) + a^a{}_i(-\vec{k})^\dagger)\exp(i\vec{k}.\vec{x}) \tag{4a}$$

and

$$^TG^a{}_{0i}(\vec{x}) = (2\pi)^{-3/2}i\int d\vec{k}\sqrt{\frac{\omega(\vec{k})}{2}}(-a^a{}_i(\vec{k}) + a^a{}_i(-\vec{k})^\dagger)\exp(i\vec{k}.\vec{x}), \tag{4b}$$

where, $\omega(k)$ is arbitrary [15] and for equal time algebra we have

$$\left[a^a{}_i(\vec{k}), a^b{}_j(\vec{k}')^\dagger\right] = \delta^{ab}\Delta_{ij}(\vec{k})\delta(\vec{k} - \vec{k}'), \tag{5}$$

with

$$\Delta_{ij}(\vec{k}) = \delta_{ij} - \frac{k_i k_j}{k^2}. \tag{6}$$

The equal time quantization condition for the fermionic sector is given as

$$[\psi^i_\alpha(\vec{x},t), \psi^j_\beta(\vec{y},t)^\dagger]_+ = \delta^{ij}\delta_{\alpha\beta}\delta(\vec{x}-\vec{y}), \tag{7}$$

where i and j refer to the colour and flavour indices [8,9]. We now also have the field expansion for fermion field ψ at time t=0 given as [8,9]

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$$\psi^i(\vec{x}) = \frac{1}{(2\pi)^{3/2}} \int \left[U_r(\vec{k}) c^i_{Ir}(\vec{k}) + V_s(-\vec{k}) \tilde{c}^i_{Is}(-\vec{k}) \right] e^{i\vec{k}\cdot\vec{x}} d\vec{k}, \tag{8}$$

where U and V are given by [16]

$$U_r(\vec{k}) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \vec{\sigma}\cdot\hat{k} \end{pmatrix} u_{Ir}; \quad V_s(-\vec{k}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \vec{\sigma}.\hat{k} \\ 1 \end{pmatrix} v_{Is}, \tag{9}$$

for free chiral fields. The above are consistent with the equal time anticommutation conditions with [16]

$$[c^i_{Ir}(\vec{k}), c^j_{Is}(\vec{k}')^\dagger]_+ = \delta_{rs}\delta^{ij}\delta(\vec{k}-\vec{k}') = [\tilde{c}^i_{Ir}(\vec{k}), \tilde{c}^j_{Is}(\vec{k}')^\dagger]_+, \tag{10}$$

In Coulomb gauge, the expression for the Hamiltonian density, $\mathcal{T}^{00}$ from equation (1) is given as [15]

$$\begin{aligned}
\mathcal{T}^{00} =&\; : \frac{1}{2} {}^T G^a{}_{0i} {}^T G^a{}_{0i} + \frac{1}{2} W^a{}_i (-\vec{\nabla}^2) W^a{}_i + g f^{abc} W^a_i W^b_j \partial_i W^c_j \\
&+ \frac{g^2}{4} f^{abc} f^{aef} W^b_i W^c_j W^e_i W^f_j + \frac{1}{2} (\partial_i f^a)(\partial_i f^a) \\
&+ \bar{\psi}(-i\gamma^i \partial_i)\psi - g\bar{\psi}\gamma^i \frac{\lambda^a}{2} W^a_i \psi :,
\end{aligned} \tag{11}$$

where $:\;:$ denotes the normal ordering with respect to the perturbative vacuum, say $| \, vac >$, defined through $a^a{}_i(\vec{k}) \, | \, vac >= 0$, $c^i_{Ir}(\vec{k}) \, | \, vac >= 0$ and $\tilde{c}^i_{Ir}(\vec{k})^\dagger \, | \, vac >= 0$. In order to solve for the operator f^a, we first note that

$$f^a = -W^a{}_0 - g \, f^{abc} \, (\vec{\nabla}^2)^{-1} (W^b{}_i \, \partial_i W^c{}_0). \tag{12}$$

Proceeding as earlier [8] with a mean field type of approximation we obtain,

$$\vec{\nabla}^2 W^a{}_0(\vec{x}) + g^2 \, f^{abc} f^{cde} \; < vac' \, | \, W^b{}_i(\vec{x}) \partial_i (\vec{\nabla}^2)^{-1} (W^d{}_j(\vec{x}) \, | \, vac' > \partial_j W^e{}_0(\vec{x}))$$

$$= J^a{}_0(\vec{x}), \tag{13}$$

where,

$$J^a_0 = g f^{abc} W^{bT}_i G^c_{0i} - g\bar{\psi}\gamma^0 \frac{\lambda^a}{2} \psi. \tag{14}$$

As noted earlier [8,11], the solution of equation (13) does not suffer from Gribov ambiguity [17]. Clearly the solution for $W_0^a(\vec{x})$ will depend on the ansatz for the ground state $|vac' >$.

We shall now consider a trial state with gluon as well as quark condensates. We thus explicitly take the ansatz for the above state as [8–11]

$$|vac' >= U_G U_F |vac >, \tag{15}$$

obtained through the unitary operators U_G and U_F on the perturbative vacuum. For the gluon sector, we have [8]

$$U_G = \exp\left(B_G^\dagger - B_G\right), \tag{16}$$

with the gluon condensate creation operator $B_G^\dagger$ as given by [8]

$$B_G^\dagger = \frac{1}{2}\int f(\vec{k}) a^a{}_i(\vec{k})^\dagger a^a{}_i(-\vec{k})^\dagger d\vec{k}, \tag{17}$$

where $f(\vec{k})$ describes the vacuum structure with gluon condensates. For fermionic sector we have,

$$U_F = \exp\left(B_F^\dagger - B_F\right), \tag{18}$$

with [9–11]

$$B_F^\dagger = \int \left[h(\vec{k}) c^i{}_I(\vec{k})^\dagger (\vec{\sigma}\cdot\hat{k}) \tilde{c}^i_I(-\vec{k})\right] d\vec{k}, \tag{19}$$

Here $h(\vec{k})$ is a trial function associated with quark antiquark condensates. We shall minimise the energy density for $|vac' >$ to analyse vacuum stability. For this purpose we first note that with the above transformation the operators corresponding to $| vac' >$ are related to the operators corresponding to $|vac >$ through the Bogoliubov transformations

$$\begin{pmatrix} b_i^a(\vec{k}) \\ b_i^a(-\vec{k})^\dagger \end{pmatrix} = \begin{pmatrix} \cosh f(\vec{k}) & -\sinh f(\vec{k}) \\ -\sinh f(\vec{k}) & \cosh f(\vec{k}) \end{pmatrix} \begin{pmatrix} a_i^a(\vec{k}) \\ a_i^a(-\vec{k})^\dagger \end{pmatrix} \tag{20}$$

for the gluon sector and

$$\begin{pmatrix} d_I(\vec{k}) \\ \tilde{d}_I(-\vec{k}) \end{pmatrix} = \begin{pmatrix} \cos(h(\vec{k})) & -(\vec{\sigma}\cdot\hat{k})\sin(h(\vec{k})) \\ (\vec{\sigma}\cdot\hat{k})\sin(h(\vec{k})) & \cos(h(\vec{k})) \end{pmatrix} \begin{pmatrix} c_I(\vec{k}) \\ \tilde{c}_I(-\vec{k}) \end{pmatrix}, \tag{21}$$

for the quark sector.

Our job now is to evaluate the expectation value of $\mathcal{T}^{00}$ with respect to $\mid vac' >$. For evaluating the same, the following formulae will be useful.

$$< vac' \mid : W^a{}_i(\vec{x})W^b{}_j(\vec{y}) : \mid vac' > = \delta^{ab} \times (2\pi)^{-3} \int d\vec{k} e^{i\vec{k}.(\vec{x}-\vec{y})} \frac{F_+(\vec{k})}{\omega(k)} \Delta_{ij}(\vec{k}), \tag{22}$$

$$< vac' \mid : {}^TG^a_{0i}(\vec{x})^T G^b_{0j}(\vec{y}) : \mid vac' > = \delta^{ab} \times (2\pi)^{-3} \int d\vec{k} e^{i\vec{k}.(\vec{x}-\vec{y})} \Delta_{ij}(\vec{k})\omega(k)F_-(k). \tag{23}$$

In the above $F_\pm(k)$ are given as [8]

$$F_\pm(\vec{k}) = \sinh^2 f(k) \pm \frac{\sinh 2f(k)}{2} \tag{24}$$

Similarly, for the quark fields we have the parallel equations given as

$$<: \psi^i_\alpha(\vec{x})^\dagger \psi^j_\beta(\vec{y}) :>_{vac'} = (2\pi)^{-3}\delta^{ij} \int \left(\Lambda_-(\vec{k})\right)_{\beta\alpha} e^{-i\vec{k}.(\vec{x}-\vec{y})} d\vec{k}, \tag{25a}$$

$$<: \psi^i_\alpha(\vec{x})\psi^j_\beta(\vec{y})^\dagger :>_{vac'} = (2\pi)^{-3}\delta^{ij} \int \left(\Lambda_+(\vec{k})\right)_{\alpha\beta} e^{i\vec{k}.(\vec{x}-\vec{y})} d\vec{k}, \tag{25b}$$

where

$$\Lambda_\pm(\vec{k}) = \pm\frac{1}{2}(\gamma^0 \sin 2h(\vec{k}) - 2(\vec{\alpha}\cdot\hat{k}) \sin^2 h(\vec{k})). \tag{26}$$

These relations will be used to evaluate the energy expectation value which is carried out in the next section.

III. EXTREMISATION OF ENERGY FUNCTIONAL AND RESULTS

We shall here proceed to evaluate the expectation value of the Hamiltonian of equation (11). However for that we note that we have to know the auxilary field contribution $(1/2)\partial_i f^a \partial_i f^a$ in equation (11) or equivalently the contribution arising out of the time like

and the longitudinal components of the gauge field. As stated earlier we shall take a mean field type of approximation [8] to solve for W_0^a field as in equation (13). Then the solution for W_0^a field in equation (13) is given as [8]

$$\tilde{W}_0^a(\vec{k}) = \frac{J^a{}_0(\vec{k})}{k^2 + \phi(\vec{k})} \tag{27}$$

where $\tilde{W}_0^a(\vec{k})$ and $\tilde{J}_0^a(\vec{k})$ are Fourier transforms of $W_0^a(\vec{x})$ and $J_0^a(\vec{x})$ and, $\phi(\vec{k})$ given through

$$\phi(k) = \frac{3g^2}{8\pi^2} \int \frac{dk'}{\omega(k')} \, F_+(k') \left(k^2 + k'^2 - \frac{(k^2 - k'^2)^2}{2kk'} \log\left|\frac{k + k'}{k - k'}\right|\right). \tag{28}$$

Using equations (11), (22), (23) and (25), we then obtain the expectation value of $\mathcal{T}^{00}$ with respect to $\mid vac' >$ as

$$\epsilon_0 \equiv \, < vac' \mid : \mathcal{T}^{00} : \mid vac' >$$
$$= C_F + C_1 + C_2 + C_3{}^2 + C_4, \tag{29}$$

where [8–10]

$$C_F = \, <: \bar{\psi}(-i\gamma^i \partial_i)\psi :>_{vac'}$$
$$= \frac{12N_f}{(2\pi)^3} \int d\vec{k} |\vec{k}| \sin^2 h(\vec{k}), \tag{30a}$$

$$C_1 = \, <: \frac{1}{2}{}^T G^a{}_{0i}{}^T G^a{}_{0i} :>_{vac'}$$
$$= \frac{4}{\pi^2} \int \omega(k) k^2 F_-(k) \, dk, \tag{30b}$$

$$C_2 = \, <: \frac{1}{2} W^a{}_i (-\vec{\nabla}^2) W^a{}_i :>_{vac'}$$
$$= \frac{4}{\pi^2} \int \frac{k^4}{\omega(k)} \, F_+(k) \, dk \tag{30c}$$

$$C_3{}^2 = \, <: \frac{1}{4}g^2 f^{abc} f^{aef} W^b{}_i W^c{}_j W^e{}_i W^f{}_j :>_{vac'}$$
$$= \left(\frac{2g}{\pi^2} \int \frac{k^2}{\omega(k)} \, F_+(k) \, dk\right)^2, \tag{30d}$$

and

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$$C_4 = <: \frac{1}{2}(\partial_i f^a)(\partial_i f^a) :>_{vac'},$$

$$= 4 \times (2\pi)^{-6} \int d\vec{k} \frac{G_1(\vec{k}) + G_2(\vec{k})}{k^2 + \phi(k)}. \tag{30e}$$

In the above,

$$G_1(\vec{k}) = 3g^2 \int d\vec{q} F_+(|\vec{q}\,|) \; F_-(|\vec{k} + \vec{q}|) \; \frac{\omega(|\,\vec{k} + \vec{q}\,|)}{\omega(|\,\vec{q}\,|)}$$

$$\times \left(1 + \frac{(q^2 + \vec{k}.\vec{q})^2}{q^2(\vec{k} + \vec{q})^2} \right), \tag{31a}$$

and, the contribution from quarks,

$$G_2(\vec{k}) = -\frac{N_f}{2} g^2 \int d\vec{q} [\sin 2h(\vec{q}) \sin 2h(\vec{k} + \vec{q})$$

$$+ 4 \frac{\vec{q} \cdot (\vec{k} + \vec{q})}{|\vec{q}||\vec{k} + \vec{q}|} \sin^2 h(\vec{q}) \sin^2 h(\vec{k} + \vec{q})] \tag{31b}$$

and $\phi(\vec{k})$ as given earlier in equation (28). As may be noted here, the contributions from the quark condensates to the energy density comes through auxiliary equation through W_0^a as well as from the quark kinetic term.

We shall now minimise the energy functional ϵ_0 of equation (29). For the same we shall take $\omega(\vec{k})$ to be of the free field form with an effective mass parameter for the gluon fields given as

$$\omega(\vec{k}) = \sqrt{k^2 + m_G^2}. \tag{32}$$

Here the gluon mass parameter m_G given through the self consistency condition [8]

$$m_G^2 = \frac{2g^2}{\pi^2} \int \frac{k^2}{\omega(k)} F_+(k) \; dk \tag{33}$$

arising from the sum of the single contractions of the quartic gluon field interaction terms of $\mathcal{T}^{00}$ in equation (11) [8,18].

The condensate functions $f(\vec{k})$ and $h(\vec{k})$ are to be determined such that the energy density ϵ_0 in (29) is a minimum. In simple cases it could be possible to solve for the condensate functions through functional differentiation [10,12]. In the present case however

the dependence of the energy density on the condensate functions is highly nonlinear and it is not possible to solve for the same through functional differentiation and equating it to zero. We therefore adopt here the alternative procedure of taking a reasonably simple ansatz for the condensate functions by parameterizing the same. We parameterize the gluon condensates as, with $k = |\vec{k}|$,

$$\sinh f(\vec{k}) = Ae^{-Bk^2/2}, \tag{34}$$

which corresponds to taking a gaussian distribution for the perturbative gluons in the non-perturbative vacuum [8]. Similarly for the function $h(\vec{k})$ describing the quark antiquark condensates we take the ansatz

$$\tan 2h(\vec{k}) = \frac{A'}{(e^{R^2k^2} - 1)^{1/2}}. \tag{35}$$

The above is a generalisation of the ansatz of ref. [11] which corresponds to $A' = 1$ and vanishes when $A' = 0$. It will be determined through energy minimisation. We note that the for free massive fermions $\tan 2h(\vec{k}) = m/|\vec{k}|$, so that the above ansatz corresponds to a momentum dependent mass given as

$$m(k) = \frac{kA'}{(e^{R^2k^2} - 1)^{1/2}}. \tag{36}$$

We may add here that such a definition of quark mass is the same as that obtained from the pole of the fermion propogator in a condensate vacuum [9,19]. In the limit of zero momentum we then have the dynamically generated mass for the quarks given as

$$m_q = \frac{A'}{R}. \tag{37}$$

The relationship between the decay constant of pion and the quark condensate function has been discussed in ref. [11] and is given as

$$\frac{N_\pi}{(2\pi)^{3/2}} \int \sin^2 2h(k)d\vec{k} = \frac{c_\pi(m_\pi)^{1/2}}{\sqrt{6}} \tag{38}$$

where $N_\pi \sin 2h(k)$ is the wave function for the pion with

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$$N_\pi^{-2} = \int \sin^2 2h(k)d\vec{k}. \tag{39}$$

With the ansatz of equation (35) we then have

$$R = \left(\frac{\sqrt{3}}{\pi c_\pi \sqrt{m_\pi}}\right)^{2/3} \cdot \left[\int \frac{A'^2 x^2 dx}{e^{x^2} + 1 - A'^2}\right]^{1/3}. \tag{40}$$

The above equation determines R as a function of A' when c_π is known. This will be used when we extremise energy along with a parallel constraint for SVZ parameter as in equation (48).

With the above ansatzes for the condensate functions the energy density ϵ_0 may now be written in terms of the dimensionless quantities $x = Rk$ and $b = B/R^2$ as

$$\begin{aligned}
\epsilon_0(A) &= \frac{1}{R^4}(I_F + I_1(A, b) + I_2(A, b) + I_3(A, b)^2 + I_4(A, b)) \\
&\equiv \frac{1}{R^4}F(A, b),
\end{aligned} \tag{41}$$

where [8–10]

$$I_F = \frac{3N_f}{\pi^2} \int x^3 dx \left[1 - \left(1 - \frac{A'^2}{e^{x^2} - 1 + A'^2}\right)^{1/2}\right], \tag{42a}$$

$$I_1(A, b) = \frac{4}{\pi^2} \int \omega(x, b)x^2 dx \left(A^2 e^{-bx^2} - Ae^{-bx^2/2}(1 + A^2 e^{-bx^2})^{1/2}\right), \tag{42b}$$

$$I_2(A, b) = \frac{4}{\pi^2} \int \frac{x^4 dx}{\omega(x, b)} \left(A^2 e^{-bx^2} + Ae^{-bx^2/2}(1 + A^2 e^{-bx^2})^{1/2}\right) \tag{42c}$$

$$I_3(A, b) = \frac{2g}{\pi^2} \int \frac{x^2 dx}{\omega(x, b)} \left(A^2 e^{-bx^2} + Ae^{-bx^2/2}(1 + A^2 e^{-bx^2})^{1/2}\right) \tag{42d}$$

and

$$I_4(A, b) = 4 \times (2\pi)^{-6} \int d\vec{x} \frac{G_1(\vec{x}) + G_2(\vec{x})}{x^2 + \phi(x)}. \tag{42e}$$

with

$$\begin{aligned}
G_1(\vec{x}) &= 3g^2 \int d\vec{x}' \left(A^2 e^{-bx'^2} + Ae^{-bx'^2/2}(1 + A^2 e^{-bx'^2})^{1/2}\right) \\
&\times \left(A^2 e^{-b(\vec{x}+\vec{x}')^2} + Ae^{-b(\vec{x}+\vec{x}')^2/2}(1 + A^2 e^{-b(\vec{x}+\vec{x}')^2})^{1/2}\right) \\
&\times \frac{\omega(|\vec{x} + \vec{x}'|)}{\omega(x')} \times \left(1 + \frac{(\vec{x}'^2 + \vec{x} \cdot \vec{x}')^2}{x'^2(\vec{x} + \vec{x}')^2}\right),
\end{aligned} \tag{43}$$

$$G_2(\vec{x}) = -\frac{N_f}{2} g^2 \int d\vec{x}\,' \left[\frac{A'^4}{(e^{x'^2} - 1 + A'^2)(e^{(\vec{x}+\vec{x}')^2} - 1 + A'^2)} + \frac{\vec{x}\,' \cdot (\vec{x} + \vec{x}\,')}{|\vec{x}\,'||\vec{x} + \vec{x}\,'|} \right.$$

$$\left. \times \left(1 - \left(1 - \frac{A'^2}{(e^{x'^2} - 1 + A'^2)} \right)^{1/2} \right) \left(1 - \left(1 - \frac{A'^2}{(e^{(\vec{x}+\vec{x}')^2} - 1 + A'^2)} \right)^{1/2} \right) \right], \qquad (44)$$

and

$$\phi(\vec{x}) = \frac{3g^2}{8\pi^2} \int \frac{dx'}{\omega(\vec{x}')} \left(A^2 e^{-bx'^2} + A e^{-bx'^2/2} (1 + A^2 e^{-bx'^2})^{1/2} \right)$$

$$\times \left(x^2 + x'^2 - \frac{(x^2 - x'^2)^2}{2xx'} log\left| \frac{x + x'}{x - x'} \right| \right). \qquad (45)$$

In the above, $\omega(x, b) = (x^2 + \mu'^2)^{1/2}$, with $\mu' = m_G R$ being the gluon mass. The self consistency condition [8] for gluon mass m_G in equation (33) yields

$$\mu^2 = \frac{2g^2}{\pi^2} \int \frac{x^2 dx}{(x^2 + \mu^2)^{1/2}} \left(A^2 e^{-x^2} + A e^{-x^2/2} (1 + A^2 e^{-x^2})^{1/2} \right), \qquad (46)$$

where $\mu = m_G \sqrt{B}$. Clearly μ' and μ are related as $\mu' = \mu/\sqrt{b}$. For given values of A and A', we first determine $R \equiv R(A\,')$ from equation (39), and then the parameter b from the SVZ parameter for gluon condensates through the equation

$$\frac{g^2}{4\pi^2} <: G_{\mu\nu}^a G^{a\mu\nu} :>_{vac} = 0.012 \, GeV^4, \qquad (47)$$

which explicitly gives [8]

$$\frac{1}{R(A\,')^4} \times \frac{g^2}{\pi^2} \left(-I_1(A, b) + I_2(A, b) + I_3(A, b)^2 - I_4(A, b) \right) = 0.012 \, GeV^4, \qquad (48)$$

so that b is a function of A and $A\,'$.

We now minimise the energy density ϵ_0 with respect to the parameters A and A' with a self consistent determination for the gluon mass as in equation (47) for different values of the QCD coupling constant α_s. We plot in curve 1 of Fig.1 ϵ_0 as a function of α_s. We note that for $\alpha_s \leq 0.62 = \alpha_c$ the condensate functions vanish, so that for the present ansatz, perturbative vacuum is stable, whereas, for $\alpha_s > \alpha_c$, we have a transition to a nonperturbative vacuum. The energy density becomes more negative with increase of coupling and becomes about -55 MeV/fm^3 when $\alpha_s = 1.4$. In curve 2 of the same figure we have plotted gluon condensate

parameter A_{min} for different couplings. In contrast to ref. [8], near $\alpha_s = \alpha_c$ this parameter has a discontinuity. In curve 3 we have plotted $\sqrt{B}$ in units of fermi as a function of coupling. The length scale here is of the order of a fermi which appears to be reasonable in the context of confinement. In curve 4, we have plotted the gluon mass parameter m_G in MeV as a function of coupling. This quantity appears to be constant and is around 300 MeV as earlier [8], and is similar to the result of Cornwall [6] where we adjust for the QCD parameter Λ or, Monte Carlo simulations of lattice calculations [20]. For such gluons QGP signals are considered for relativistic heavy ion collisions [18].

In Fig. 2 we have plotted different characteristics of the quark condensate functions. In curves 1 and 2 we have plotted the condensate parameters A'_{min} and R respectively. They seem to increase as we approach α_c from above. We have plotted in curve 3 the quark condensate value given as

$$< -\bar{\psi}\psi >^{1/3} = \Big[\frac{12}{(2\pi)^3} \int d\vec{k} \sin 2h(\vec{k}) \Big]^{1/3}. \tag{49}$$

The condensation effect increases with coupling as expected. In curve 4 we have plotted the dynamically generated quark mass m_q of equation (37). We may in fact identify the same as the parallel of constituent mass of quarks as obtained from chiral symmetry breaking.

We may remark that for $\alpha_s < \alpha_c$, any nonzero trial functions make ϵ_0 positive, which shows that for the present ansatz the perturbative vacuum is stable.

IV. DISCUSSIONS

We have considered here destabilisation of perurbative vacuum in QCD as a function of coupling constant through an explicit construction. We have taken an ansatz for nonperturbative vacuum with trial functions both for quark and gluon condensates. With the QCD Hamiltonian for quarks and gluons satisfying equal time algebra, we then do a minimisation of the energy density with the constraints that the pion decay constant and the SVZ parameters are correctly reproduced. Minimisation of the energy density over remaining

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parameters yields that vacuum with condensates is the preferred configuration when $\Phi \, \alpha_s$ is greater than 0.62. In order to relate the vacuum structure of chiral symmetry breaking to c_π, we exploit the results of Ref. [9–11].

It shall be amusing to consider the present results in the context of Ref. [11] where *no* extremisation was done. We find that for $\alpha_s = 1.28$, $A'_{min} \simeq 1$, and that the pion charge radius gets correctly reproduced. In fact, with $A' = 1$, we have [11]

$$R^2_{ch} = R^2_1 + R^2_2, \tag{50}$$

where

$$R^2_1 = \frac{N^2_\pi}{4} \int |\vec{\nabla} \sin 2h(k)|^2 d\vec{k} \tag{51}$$

and

$$R^2_2 = \frac{N^2_\pi}{16} \int \sin^2 2h(k) \Big[k^2 R^4 \tan^2 2h(k) + \frac{4(1 - \sin 2h(k))}{k^2} \Big] d\vec{k} \tag{52}$$

In the above N_π is given in equation (39). With $R \simeq 0.96$ fm, we then obtain that $R_{ch} \simeq 0.65$ fm, which agrees with the experimental value of 0.66 fm [21] The new feature here is that we are able to relate the QCD coupling constant with low energy hadron properties.

We may also note that the averge number of perturbative gluons or quarks in such a condensate vacuum is given as

$$N_G =< vac'|a^a_i(\vec{z})^\dagger a^a_i(\vec{z})|vac' >= 2 \times 8 \times \frac{1}{2\pi^3} \int \sinh^2 f(\vec{k}) d\vec{k}. \tag{53}$$

and,

$$N_q = \frac{12}{(2\pi)^3} \int \sin^2 h(\vec{k}) d\vec{k}$$
$$= \frac{1}{R^3} \frac{3}{\pi^2} \int x^2 dx \Big[1 - \Big(1 - \frac{A'^2}{e^{x^2} - 1 + A'^2} \Big)^{1/2} \Big]. \tag{54}$$

We then obtain that for example when $\alpha_s = 1.28$ corresponding to correct charge radius of pion, $N_G = 2A^2/(\pi B)^{3/2} \simeq 0.233/\text{fm}^3$ and that $N_q \simeq 0.085/\text{fm}^3$. We also note that

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for $\alpha_s = 1.28$, the "bag pressure" is given as $(-\epsilon_0)^{1/4} = 140$ MeV, which appears to be in general agreement with the phenomenological value of this parameter.

We thus find that a constrained energy minimisation of QCD Hamiltonian can lead to both gluon and quark condensates with appropriate chiral symmetry breaking. Even we can have simultaneously both the charge radius and pion decay constant, features of chiral symmetry breaking, getting related to the QCD coupling constant. The method in fact seems to open up possibilities of linking up the (mysterious) vacuum structure of QCD to (common place) hadronic properties in an unexpected manner.

We have focussed our attention on the structure problem. It will be desirable to include the dynamical effects in the light quark sector. Also, chiral symmetry breaking is important in the strange quark sector, and calculations adding condensates of the same can be carried out. Besides technical difficulty, this has the additional feature of Lagrangian mass being of the same order as dynamical mass and may need nontrivial extension of the technology. Work in this direction are in progress.

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Figure Captions

Fig.1: In curves 1,2,3 and 4, energy density ϵ_0 (in units of 100 MeV/fm^3), A_{min} , $\sqrt{B}$ (in units of fm) and effective gluon mass m_G (in units of 200 MeV), are plotted respectively as functions of strong coupling constant α_s.

Fig.2: In curves 1,2,3 and 4, A'_{min},R (in units of fm), the quark condensate $(- <\bar{\psi}\psi>)^{1/3}$ (in units of 100 MeV) and dynamically generated quark mass m_q (in units of 200 MeV), are plotted respectively as functions of the coupling constant. α_s.